UNIT 6: PULLING THE THREADS TOGETHER

6.1 Constructing and Evaluating Theories: the Art and the Craft

We expect that by now, you would have an intuitive understanding of how a theory is constructed. Let us state that understanding explicitly.

The first step when we construct a theory of X is to write down a set of statements that we already know about X. This is what we called a ***description*** of X.

Among the statements, we select some as premises, and others that we can derive as logical consequences of those premises, we take as conclusions. Having done this, we provide proofs for the conclusions, by demonstrating how they are derived. This is what we have called the **P**(remises)-**D**(erivation)-**C**onclusion structure.

We make this move with caution. It is tentative, because in trying to derive the conclusions from the premises, we may change their status. It may turn out that some of the statements that we initially viewed as conclusions cannot be derived from the premises, so we will need to take them also as premises. As a consequence, some of the statements that we had thought of as premises may become conclusions. You had a taste of this kind of a situation in §4.3, in our discussion of the definitions of parallel lines vs. the theorems that follow logically.

Having separated the statements into premises and conclusions, we further separate the premises, equally tentatively, into definitions and axioms. This sifting is necessary because it shapes our conceptualisation. However, it would not affect our proofs, since whether we take a statement as an axiom or a definition does not have any effect on our derivation.

The next step is to evaluate our proofs. Does each step in our derivation follow logically from the preceding steps and the premises? In other words, have our attempts at proofs given us valid proofs? If there are steps that make our proofs invalid, we need to modify our premises, or unearth statements that we have been implicitly assuming, and formulate them as explicit premises. This is likely to increase the number of premises.

Alongside adding our implicit assumptions as explicit premises, we might also add to our list of ***conjectures***. These are the statements that we think are true of X, but have not proved them. Every time we add a conjecture, we repeat the steps outlined above, so that the process is thorough.

As mentioned earlier, for theories outside of mathematics, we need one other component. Scientific theories, for example, require us to discover and establish observational generalisations, and show that

- (i) our theoretical conclusions follow from (are correct predictions of) these generalisations, and
- (ii) they can successfully explain the asymmetries that these generalisations point to.

Theories of other kinds, including conceptual theories, ethical theories, and theories of value require us to satisfy similar external conditions that involve equivalents of observational generalisations.

6.2 Abstracting

Geometry is a mathematical theory concerned with shapes, their relations and relative arrangements in space. It is ultimately rooted in our visual experience of the world. When we compare a sheet of A4 size paper on a table with a DVD next to it, we distinguish between the shapes of these objects as rectangular vs. circular. When we say rectangular and circular, we are focusing on the shape. The objects — the sheet of paper and the disk — have other properties such as colour, weight, rigidity, opacity, and so on. But geometry ignores these aspects, and extracts the relevant attributes of the objects into abstract entities. These ***abstract objects***, which we call *circle, rectangle*, and so on, do not have colour, weight, rigidity, opacity, and the like. So a spherical apple, for example, is edible, but a sphere is not.

Such abstraction, which isolates and studies properties of concrete objects in the world, and transforms them into abstract attributes or abstract objects, is not restricted to mathematical theories. Scientific theories are also built out of such abstractions.

6.3 Generalising and Specialising

Imagine this. We take a sample of ten polygons, and build descriptions of each of them. These descriptions would be the equivalents of what are called 'data points' in scientific theories. Based on these descriptions, we come up with a set of conjectures. We now examine three more polygons, and check if our conjectures hold on them. And then we examine five more polygons. We go on doing this until we can find no ***counterexamples*** to our conjectures. Having satisfied ourselves that there are no counterexamples, we can now proceed to prove the conjectures.

What we have described above is the ***process of generalisation*** from a sample of polygons to the population of polygons. This methodological strategy of generalising from a sample to a population is relevant in any form of collective inquiry, including scientific inquiry.

Needless to say, in the process of generalising from the sample to the population, we might discover counterexamples. When faced with counterexamples, we may either abandon our original conjecture as false, or see if it can be saved by restricting the scope of the conjecture to a sub-population. For instance, we might find that all our counterexamples involve polygons with concave angles, in which case we might restrict the scope of our conjecture to convex polygons. We may have to restrict the scope even further, to regular polygons, and so on. Such a move involves the ***process of specialisation***.

Both generalisation and specialisation are needed in theories outside mathematics as well. We might originally propose a conjecture on vertebrates, but we may find that it applies to animals in general, in which case we generalise it. Alternatively, we may discover counterexamples and may need to restrict the scope to mammals, in which case we specialise it.

6.4 Reasoning, Predicting, Explaining and Proving

As we have seen, theories of geometry are ***axiomatic systems***. This is true of all mathematical theories. In such a theory, a conjecture is ***proved*** by deriving it from the premises, using deductive ***reasoning***. So mathematical theories involve constructing knowledge through pure reasoning, combined with imagination, insight, and intuition.

Axiomatic systems are subject to the conditions of coherence. One of the conditions of coherence is the absence of logical contradictions internal to the system. In addition, coherence also includes the conditions of ***logical connectedness***, ***generality*** (widest range of conclusions), and ***simplicity*** (fewest possible premises — also called *parsimony*, or *Occam's Razor*).

Since scientific theories have an axiomatic component, they are also subject to the conditions of coherence. The scientific equivalents of mathematical theorems, as we have seen, are ***predictions***: the logical consequences deduced from the premises of the theory. But as pointed out earlier, scientific theories have the additional requirement that the predictions must agree with the observational generalisations. That translates as the requirement that not only must theories make predictions, but those predictions must be correct, where 'correct' means logically consistent with the observational generalisations.

An important condition that we must point out at this juncture is that it is not enough for the predictions to be corect. They must also explain the asymmetries in observational generalisations. In other words, scientific theories simultanously respond to the questions, "Is it true?" (rational justification), and "Why is it true?" (explanation, and the understanding that comes from that explanation). Explanation is central to scientific theories.

In the latter function, that of explanation, mathematical theories also don't just predict, but also explain the asymmetries in statements that are accepted as true.

Let us take a few examples. Here are a few statements that we take to be 'true statements' (TS). For each of them, we need to ask: "Why is this so?"

TS1: No triangle can have a reflex angle.
Non-triangle polygons can have reflex angles.

TS2: An equilateral triangle is an equiangular triangle and vice versa.
Non-triangle polygons do not exhibit this pattern,

TS3: An angle in a triangle cannot be varied without simultaneously varying the length of at least one of its sides.
Non-triangle polygons do not exhibit this behaviour,

Mathematical proofs that demonstrate the truth of the above statements also help us understand the asymmetries in TS1-3 by providing explanations for them.

6.5 Transdisciplinary Epistemology

The concepts and strategies of theory construction illustrated in this monograph were introduced in the specific context of geometry, though we occasionally demonstrated their usefulness in theory construction in biology.

These concepts and strategies are not restricted to geometry and biology: they play an important role in the construction and evaluation of theories in all domains of research, ranging from mathematics, and the physical-biological-human sciences, to the humanities. We will therefore refer to them as ***transdisciplinary*** concepts and strategies, using *trans-* in the sense that they cut across specialised disciplines, discipline groups and fields, and exist at a more abstract level.

To take an example of what we mean by transdisciplinary, the so-called *theory of classical mechanics* is specific to the highly specialised field that studies gravity and motion within the discipline called physics. However, the concept of ***theory*** is a trans-disciplinary one. It is in this sense that we talk about transdisciplinary concepts.

Take another example. The concepts of gravity, velocity, and acceleration are specific to physics, and the concept of skeletal structure, and of cells being composed of biomolecules, is specific to biology. But the concepts of ***compositionality***, ***structure***, and very concept of ***concept*** are transdisciplinary.

In §1.5.3, we made a distinction between ***axioms*** that are specific to a particular field or discipline, and ***general axioms*** that are part of all academic knowledge. General axioms are transdisciplinary. We may make the same distinction among definitions. The definition of force as *that which causes a change in the velocity of an inanimate entity* is specific to Physics. But the definition of force as *that which causes change* is a transdisciplinary definition, extendable to the animate, even human domains.

To repeat what we said in §3.3:

Transdisciplinary relations that appear in theories across disciplines include:

- Subcategorization: *x* is A SUBCATEGORY OF *y*.
- Compositionality: *x* is COMPOSED OF *y, z, …* (Variants: is MADE UP OF / is A CONSTITUENT OF, is DECOMPOSABLE INTO, …)
- Ordering: *x* is ORDERED PRIOR TO *y*. (Variants: is RANKED HIGHER THAN; PRECEDES)
- Logical consequence: *x* is a LOGICAL CONSEQUENCE OF *y*.
- Logical contradiction: *x* LOGICALLY CONTRADICTS *y*.
- Equality: *x* IS EQUAL TO *y*. (Variant: IS EQUIVALENT TO, IS AN ANALOGUE OF, IS A HOMOLOGUE OF, …)
- Correlation: *x* CORRELATES WITH *y*.
- Causation: *x* CAUSES *y*
- Instantiation: *x* is AN INSTANCE OF *y*. (Variants: is A MEMBER OF set/category y, is AN EXAMPLE OF y, is A SAMPLE OF y, …)
- Negation: x is THE NEGATION OF *y*. (Variant: is THE OPPOSITE OF.) (Also see logical contradiction)

We hope that the journey through geometry that we have undertaken so far has given you a glimpse into the transdisciplinary epistemology of academic knowledge, and has planted seeds that can develop the capacity to engage in research in any domain of academic knowledge.

6.6 Admissible Sources of Knowledge

The first paragraph of the introductory chapter of the book, *The Works of Archimedes*, by T L Heath begins as follows:

> "A LIFE of Archimedes was written by one Heracleides, but this biography has not survived, and such particulars as are known have to be collected from many various

> sources. According to Tzetzes he died at the age of 75, and, as he perished in the sack of Syracuse (B.C. 212), it follows that he was probably born about 287 B.C."

Notice the use of "it follows that," in the second sentence. The phrase signals the strategy of ***reasoning*** that forms of the core of ***rational inquiry***. We have discussed the methodological strategies of rational inquiry at length in the previous Units. Let us state the premises and the conclusion of the passage explicitly:

> Premise 1: Archimedes died at the age of 75
> Premise 1: Archimedes died in 212 BCE.
> Conclusion: Archimedes was born in 287 BCE.

Is this conclusion true?

We can answer that question as follows:

> If premises 1 and 2 are true, and the derivation of the conclusion from the premises is valid, then it is true that Archimedes was born in 287 BCE.

This answer tells us that there are two conditions for our accepting the truth of the conclusion:

> Condition A: The premises must be true.
> Condition B: The derivation of the conclusion from the premises must be valid.

How do we know that the premises are true? The answer is:

> Premises 1 and 2 are asserted by Tzetzes.

But why should we believe that what Tzetzes said is true? An Internet search for the name Tzetzes leads us to John Tzetzes, a Byzantine poet and grammarian who is known to have lived at Constantinople in the 12th century. T L Heath is making the assumption that John Tzetzes' ***testimony*** is a reliable ***source of knowledge*** to conclude that Archimedes was born in 287 BCE.

Relying on written testimonies by previous authors is an important methodological strategy of human history, in domains where written records are available. A similar methodological strategy is used in trials in the criminal court: spoken testimonies of eye witnesses and of expert witnesses are taken as reliable sources of knowledge.

You can see that spoken and written testimonies are not admissible as sources of knowledge in the physical and biological sciences. So, the fact that *Isaac Newton says*:

> The gravitational attraction between two bodies is directly proportional to the product of their masses, and indirectly proportional to the square of the distance between them,

does not allow us to conclude that the proposition:

> "The gravitational attraction between two bodies is directly proportional to the product of their masses, and indirectly proportional to the square of the distance between them,"

is true.

Similarly, the fact that *Charles Darwin says*:

> All existing and extinct life forms on the earth evolved from unicellular ancestors,

does not allow us to conclude that the proposition:

> "All existing and extinct life forms on the earth evolved from unicellular ancestors,"

is true. Expert testimonies are admissible sources of knowledge in criminal trials and in human history, but not in the physical and biological sciences.

What constitutes an admissible source of knowledge in scientific inquiry? The answer is: observational reports. An observational report is an eyewitness testimony.

6.7 The Nature of Truth in Mathematics

What is the nature of premises in mathematics? What are the kinds of premises that are accepted as sources of knowledge in mathematics?

In §4.2, we saw an example of the contrast between the Euclidean axiom that every finite line, however small, has infinitely many points, and the non-Euclidean axiom that every finite line has a finite number of points, such that the length of a line is the number of points it contains. If we adopt the Euclidean axiom, we deduce the conclusion that every line is bisectable. But if we adopt the alternative axiom, we deduce the conclusion that there exist lines which cannot be bisected.

These theorems are logically contradictory. So, if they were from the same theory, rationality demands that we reject at least one of them as false. But unlike the axioms (laws, constraints) in science, truth and falsehood are not properties of mathematical axioms. Why is that so?

This is because scientific theories are about the particular world we live in, but mathematical theories are about logically possible imagined worlds. And there can be many such worlds. So all that we can say is this:

> In a world in which the Euclidean axiom of the number of points in a line is true, the theorem that every line is bisectable is true.

But: In a world in which the non-Euclidean axiom of the number of points in a line is true, the theorem that there exist lines that cannot be bisected is true.

There is no logical contradiction between the two because they are about two different worlds.

Similar remarks apply to the geometry of flat surfaces and of spherical surfaces. You might have heard that Euclidean two-dimensional geometry is a geometry of flat surfaces, while Riemannian two-dimensional geometry is a geometry of spherical surfaces. The difference between them is:

IN A FLAT SURFACE GEOMETRY	IN A SPHERICAL SURFACE GEOMETRY
No straight line, regardless of how far it is extended, can meet itself.	Every straight line when extended meets itself.
No two straight lines can intersect at two distinct points.	Any two straight lines when extended intersect at two distinct points.
The sum of angles in a triangle is two right angles.	The sum of angles in a triangle is more than two right angles, and can be upto three right angles.

All this shows that the truth of a mathematical theorem is relative to the theory that it is a part of. Hence, mathematical truths are of the form:

> If such and such premises are true, such and such conclusions are also true.

Are the premises true? Mathematics has nothing to say about that. This is a fundamental difference between mathematical and scientific truths.