UNIT 2: A THEORY OF QUADRILATERALS

2.1 Squares

Here are a few examples of Squares.

Figure 2-1

What are the properties of Squares? Let us make a list:

1) Properties of Squares

a) A Square has four angles.

b) A Square has four straight lines.

c) The size of any angle in a Square is equal to that of every other angle in that Square.

d) The length of each line is equal to that of every other line in that Square.

e) In a Square, the sum of the sizes of angles is equal to four times a right angle.

f) The area of a Square is equal to the product of the lengths of any two of the sides of that Square.

g) The square of the length of the diagonal of a Square is equal to twice the sum of the square of the length of any of its sides.

Exercise 1

a) Based on your current knowledge of geometry, add to the list in (1).

b) Do an Internet search to expand your list even further. For instance, try these:

"Properties of a rhombus, a rectangle, and a square"
(https://www.youtube.com/watch?v=3i2yp-lI_V4)

"Theorems dealing with rectangles, rhombuses, and squares"
(https://mathbitsnotebook.com/Geometry/Quadrilaterals/QDRectangle.html)

c) Based on your knowledge of geometry, make a list of the properties of Rectangles, Rhombuses, and Parallelograms. Drawing on the way we put together the properties of RATs, ETs and Triangles, try to put together the properties of Squares, Rectangles, Rhombuses and Parallelograms. Having done that, set up statements of subcategories among them in such a way that using the General Axiom (5) in Unit 1, you can deduce as many properties of these geometrical objects as possible from a small number of premises.

2.2 Triangles and Quadrilaterals

Chances are that the exercises in §2.1 led you to propose that Squares, Rectangles, Rhombuses and Parallelograms are subcategories of Quadrilaterals.

Did you, by any chance, also think of proposing that Squares are subcategories of Rectangles? That Rectangles are subcategories of Parallelograms? That Squares are also subcategories of Rhombuses? If you didn't happen to think of these possibilities, reflect on them now and write down your subcategorization statements.

Having explored the ***properties*** of both Triangles and Quadrilaterals, we can now look at the ***relations*** between them. For instance, is it possible to divide a Square into two Triangles? If it is, what kind of Triangle do you get? RATs? ETs? Isosceles Triangles?

Is it possible to put together one or more Triangles into a Square? Any category of Triangles?

Explore these relations, and make a list of the statements that express them. Then try to derive the relations from the premises you have already come up with.

Now consider Pentagons. Do you see a set of shared properties in the categories of Triangles, Squares, and Pentagons? Try to formulate statements that describe what they have in common.

In (1), we stated a number of properties of Squares. Suppose we delete one or more of these properties in (a)-(d). Would you then get figures which are not Squares? What would their categories be? Spend a few minutes reflecting on this question.

2.3 Triangles, Quadrilaterals, Pentagons, Hexagons …

It is time to write descriptions of the categories of geometric figures that we have been investigating.

In Unit 1, we saw a number of repetitions in the descriptions of RATs, ETs, and Triangles. To eliminate these repetitions, we did the following:

- ~ treat RATs and ETs as subcategories of the category of Triangles;
- ~ set up a General Principle, like (5), on the subcategories of a category;
- ~ remove the redundant statements in the descriptions of RATs and ETs; and
- ~ derive those predictable statements from the premises.

Can you try to use the same methodological strategy on the descriptions of the categories of geometric figures that you have been investigating, and integrate all of them into a single theory?

Exercise 2

a) Do a Internet search for crows, vultures, and birds. For each category, write a set of sentences to describe the anatomy, along the lines illustrated in (1), (4) and (6). Do not go beyond what you judge to be ten most important points. Otherwise, this exercise will be way too time consuming.

b) Try to group the statements for each category into premises and conclusions in such a way that the number of premises is minimised. the premises are the smallest possible in number.

c) Make sure that you have a derivation for each conclusion.

d) Separate the premises into definitions and axioms.

e) Separate the axioms into General Axioms and Discipline-Specific Axioms.

Exercise 3

a) Do a google search for the vertebra of crows, vultures, and birds. For each category of vertebra, write a set of sentences to describe the structure of the vertebra. Do not go beyond what you judge to be ten most important points. Otherwise, this exercise will be way too time consuming.

b) Try to group the statements for each category of vertebra into premises and conclusions in such a way that the number of premises is minimised.

c) Make sure that you have a derivation for each conclusion.

d) Separate the premises into definitions and axioms.

e) Separate the axioms into axioms into General Axioms and Discipline Specific Axioms.

Exercise 4

a) Do a google search for the vertebra of mice, bats, and mammals. For each category of vertebra, write a set of sentences to describe the structure of the vertebra. Do not go beyond what you judge to be ten most important points. Otherwise, this exercise will be way too time consuming.

b) Try to group the statements for each category of vertebra into premises and conclusions in such a way that the number of premises is minimised.

c) Make sure that you have a derivation for each conclusion.

d) Separate the premises into definitions and axioms.

e) Separate the axioms into General Axioms and Discipline Specific Axioms.

Exercise 5

In Exercise 2 we constructed a theory of the categories of organisms. In Exercises 3 and 4, in contrast, we constructed a theory of the categories of a particular organ (vertebra) in these categories of organisms. Think of a way to construct a single theory that includes categories of both organisms and categories of organs.

Exercise 6

In Aristotle's categorisation of animate entities, the first division was human vs non-human. The category of non-human was further categorised as animal vs. plant. In the classification proposed by Carl Linnaeus, on the other hand, the first division was animal vs. plant. Humans came under animals. A third possible categorisation would be human, animal, and plant.

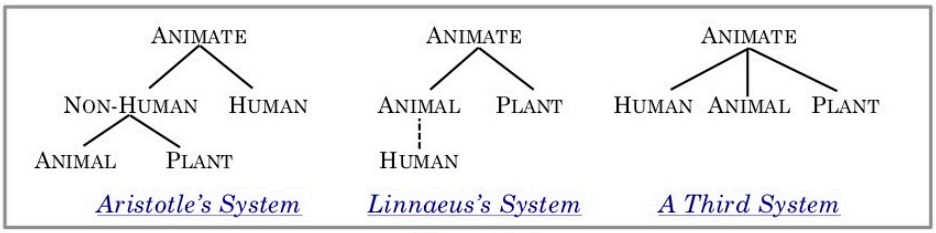

Figure 2-2

For a theory of animate entities that maximises predictions and minimises premises, which of these systems would you choose? State your reasons, drawing on what you have learnt so far on the strategies and norms of theory construction-and-evaluation.

2.4 Concluding Remarks

In Unit 1, we were introduced to the how the art and craft of theory construction by separating what is predictable from what is not predictable in descriptions, deducing what is predictable from what is not predictable. This methodological strategy is accompanied by the categorisation of the premises as definitions, and the categorisation of axioms into General Axioms and Discipline-Specific Axioms.

In Unit 2, we extended what we leant in Unit 1 to construct at theory that integrates theories of Parallelograms, Rhombuses, Rectangles and Squares. We also practiced the methodology of theory construction to construct a theory of insects, a theory of birds, and a theory of vertebra.

Our decisions on categorisation, as well as on what should be treated as premises and what should be treated as predictions, are based on the norm of

> *minimising premises and maximising predictions.*

When we are faced with the choice between competing theories (including competing categorisations and competing premise-prediction pairings), we choose that option which minimises premises and maximising predictions.

In Unit 3, we will integrate theories of Triangles, Quadrilaterals and Pentagons to construct a theory of Polygons, also covering other Polygons like Hexagons, Septagons, and Octagons. We will then proceed to Lines and Points in Unit 4.