UNIT 1: A THEORY OF TRIANGLES

1.1 Right-Angled Triangles

Here are a few examples of Right-Angled Triangles.

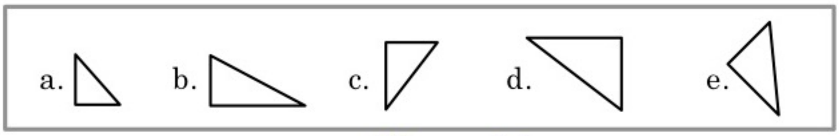

Figure 1-1

What are the properties of Right Angled Triangles? Let us make a list:

1) Properties of Right-Angled Triangles (RATs)

a) An RAT has three angles.

b) An RAT has three straight lines.

c) One of the angles of an RAT is a right angle.

d) The area of an RAT is equal to half the product of the lengths of the two sides adjacent to the right angle.

e) The length of the square of the side opposite the right angle is equal to the sum of the squares of the lengths of the other two sides.

1.2 Equilateral Triangles

Here are a few examples Equilateral Triangles:

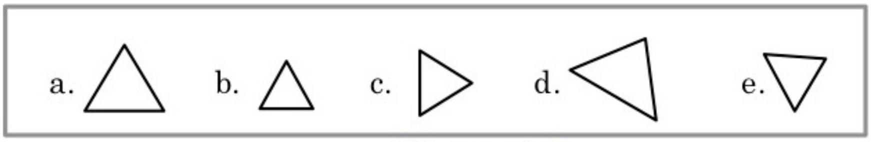

Figure 1-2

Once again, let us make a list, this time of the properties of ETs:

2) Properties of ETs:

a) An ET has three angles.

b) An ET has three straight lines.

c) In an ET, the size of any angle is equal to the size of any other angle.

d) The area of an ET is equal to half the product of the lengths of any one of the sides and the length of the perpendicular from the opposite angle to that side.

e) In an ET, the length of any one side is equal to the length of any other side.

At this point, you might experience a mild discomfort. In (2a-b), we are repeating what we said about RATs in (1a-b) as properties of ETs. While we agree that these statements are true, do we need to repeat them? How do we avoid this unnecessary repetition?

1.3 Triangles

One solution is to begin by noting that RATs and ETs are both Triangles. They (RATs and ETs) are subcategories of the category 'Triangle'. Examples of triangles include not only those in Figs. 1-1 and 1-2, but also those in Fig. 1-3:

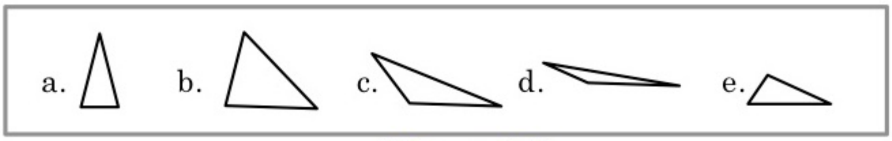

Figure 1-3

What about the properties of Triangles? Here they are:

3) Properties of Triangles

a) A Triangle has three angles.

b) A Triangle has three straight lines.

c) In a Triangle, the sum of the angles is equal to twice the sum of a right angle.

d) The area of a Triangle is equal to half the product of the length of any one of the sides, and the length of the perpendicular from the opposite angle to that side.

e) In a Triangle, the length of a side increases as the length of the perpendicular to that side from the opposite angle increases, provided the angles remain unchanged.

We now see that (1a-b) and (2a-b) follow from (3a-b). So given (3a-b), repeating the statements as (1a-b) and (2a-b) is unnecessary, or ***redundant***.

1.4 Derivation

How exactly do the statements in (1a-b) and (2a-b) follow from (3a-b)? What is the general principle that allows us to derive (1a-b) and (2a-b) from (3a-b)?

We had noted earlier that:

4) The categories of RATs and ETs are sub-categories of Triangles.

Given (4), we can derive (1a-b) and (2a-b) from (3a-b) if we postulate the following general principle:

5) **General Principle**:
The properties of a category are inherited by their subcategories.

The derivation given below illustrates the application of this general principle.

6) **Derivation**

A Triangle has three angles. (3a)

The categories of RATs and ETs are sub-categories of Triangles. (4)

The properties of a category are inherited by their subcategories. (5)

Hence, an RAT and an ET have three angles. (1a), (2a)

Exercise 1

Derive (1d) and (2d) from (3d).

1.5 The Structure of Theories

Derivations like the one illustrated above, and the logical structure that allows for such derivations, are the hallmarks of theories, distinguishing them from mere descriptions. As far as we know, Euclid in Ancient Greece and Panini in Ancient India were the people who originally developed theories in the sense of the term 'theory' in mathematical and scientific inquiry. Panini's work, called *Ashtadhyahyi*, is not accessible to students, but the English translation of Euclid's Elements is accessible to those who are willing to struggle with it. It is downloadable at:

http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf

but only after completing *Constructing Theories of Geometry*.

1.5.1 Derivations

A derivation is a sequence of steps of reasoning. What makes derivations possible are a set of assumptions that we take as the premises of an argument. Given these premises, the derivation yields a conclusion.

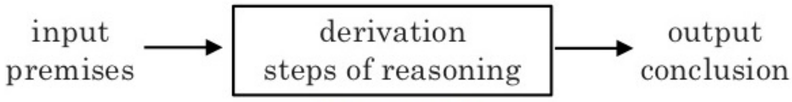

Figure 1-4

1.5.2 Definitions, Postulates, and Common Notions

Euclid distinguishes three kinds of premises: definitions, postulates, and common notions.

The ***definition*** of a concept is a statement that tells us what comes under that concept and what doesn't. For example, a definition of even numbers tells us which integers are even and which ones are not. A definition of mammals tells us which organisms are mammals and which ones are not. A definition of democracy tells us which systems of decision making are democratic and which ones are not.

Here are some examples of definitions in geometry:

Right Angle (DEF): An angle is a Right Angle if and only if it is formed by two lines that are perpendicular to each other.

Obtuse Angle (DEF): An angle is an Obtuse Angle if and only if it is formed greater than a right-angle.

Acute Angle (DEF): An angle is an Acute Angle if and only if it is less than a right-angle.

What, then, is the definition of 'perpendicular', that appears in the definition of a Right Angle?

Perpendicular (DEF): Line A is perpendicular to line B if and only if, when A stands on B and the adjacent angles are equal.

Euclid used the term ***postulate*** to denote premises that are specific to geometry. Given below are examples of postulates:

Postulate 1: Given any two distinct points, one and only one straight-line exists between them.

Postulate 2: Any finite straight-line can be extended indefinitely as a straight line.

The term ***common notion*** in Euclid denotes premises that apply to mathematics in general, not just to geometry. Here are examples of common notions:

Common Notion 1: Things that are equal to the same thing are also equal to one another.

Common Notion 2: If equal things are added to equal things, then the wholes are equal.

We will adopt the essence of the Euclidean distinction between postulates and common notions. But we will call them ***axioms***, and distinguish two categories of axioms: ***general axioms*** and ***discipline-specific axioms***.

1.5.3 Axioms and Axiomatic Systems

We will use the term ***General Axioms*** to denote those axioms that are relevant in all disciplines and discipline groups — whether geometry, number theory, mathematics, astronomy, physics, the physical sciences, evolutionary biology, developmental biology, organismic biology, cognitive science, linguistics, sociology, anthropology, economics, history, or philosophy. The General Principle in (5) is an example of such a General Axiom. Whether the common notions that Euclid set up fall under General Axioms that hold on all domains of research, or are specific to a particular kind of research, is an issue that we will pick up later.

Euclid saw axioms as 'self-evident truths'. However, subsequent develop-ments in mathematics show that they are ***assumptions***. This means that they are not self-evident truths, but premises that are set up as starting points for particular theories.

A system of knowledge that exhibits the property of deriving conclusions from axioms and definitions is called an ***axiomatic system***. Any fully fleshed-out explicit theory, whether in mathematics, the physical-biological-human sciences, or the humanities includes an axiomatic system.

Exercise 2

a. Try to group the statements in (1) (2) and (3) into premises and conclusions in such a way that the number of premises is minimised. Derive (1d) from (3d)

b. Make sure that you have a derivation for each conclusion.

c. Separate the premises into definitions and axioms.

d. Separate the axioms into axioms into General Axioms and Discipline-Specific Axioms.

Note: The term 'discipline-specific' in (d) is a bit murky because we do not have a clear idea of what a discipline is and what a group of disciplines is. E.g. is molecular biology a discipline or a field within a discipline called life sciences? Is zoology a discipline or a field with life sciences? Do the study of society (covering human sociology and non-human sociology form a single discipline? If they are two disciplines, do they constitute a single discipline group?) Granted this murkiness, it still useful to invest some time on (d).

1.6 Theories vs. Descriptions

The statements illustrated in (1), (2) and (3) are descriptions, not theories. A description is a collection of statements about a given entity. Thus, the statements in (7) are part of a description of one of the members of a human being called Zeno:

7)
- a. Zeno is a human being.
- b. Zeno is an adult.
- c. Zeno is a living organism.
- d. Zeno has eukaryotic cells.
- e. Zeno has two eyes.
- f. Zeno has a mouth.
- g. Zeno has a heart.
- h. Zeno has vertebrae.
- j. Zeno has bones.
- k. Zeno is male.
- l. Zeno is 180 cms tall.
- m. Zeno weighs 82 kilograms.
- n. Zeno is diabetic.
- o. Zeno is prone to depression.
- p. Zeno works as a banker.
- q. Zeno has two siblings.
- r. Zeno is married to Athena.
- s. Zeno has three children.

And the statements in (8) are part of a description of the human species:

8)
- a. Humans are living organisms.
- b. Humans have eukaryotic cells.
- c. Humans have two eyes.
- d. Humans have a mouth.
- e. Humans have a heart.
- f. Humans have vertebrae.
- g. Humans have bones.

An important characteristic of a theory is that it separates what is predictable from what is idiosyncratic. By 'predictable', we mean something that can be inferred from some other information. Thus, we can predict that Zeno has eukaryotic cells (7d) from the statements that Zeno is a human being (7a) and humans have eukaryotic cells (8b). A theory separates these two kinds of statements by deriving what is predictable from the premises (axioms and definitions) of the theory.

Exercise 3

- a. Do a google search for butterflies, ants, and insects, and for each category, write a set of sentences to describe the anatomy, along the lines illustrated in (1), (2) and (3). Do not go beyond what you judge to be ten most important points. Otherwise, this exercise will be way too time consuming.
- b. Try to group the statements for each category into premises and conclusions in such a way that the number of premises is minimised.
- c. Make sure that you have a derivation for each conclusion.
- d. Separate the premises into definitions and axioms.
- e. Separate the axioms into General Axioms and Discipline-Specific Axioms.

In the Units that follow, we will learn how to construct and evaluate theories by separating what is predictable from what is not predictable, and deducing what is predictable (conclusions) from what is not predictable (premises).

While learning the rudiments of this methodological strategy, we will also learn a number of other strategies and techniques of theory construction, and be introduced to some of the norms governing theory construction, and knowledge construction in general.