



CONSTRUCTING THEORIES:

A CASE STUDY IN GEOMETRY
WITH AN EXCURSION INTO BIOLOGY

K P Mohanan and Tara Mohanan



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September 2021

This book is an attempt to introduce the reader to the art and craft of the construction (and evaluation) of theories. While we use geometry as the terrain for practice, our focus is on the methodological strategies and techniques of value in all domains of theory construction, not on the ‘facts’ of geometry that students learn in textbooks.

We expect that the units in the book will be useful for researchers and research students across disciplines, but have also tried to make the content and the style accessible to high school students.

NOTE: If you find any errors in these pages, please alert us at:
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